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INSTABILITY OF TWO ROTATING OLDROYDIAN VISCOELASTIC SUPERPOSED FLUIDS THROUGH POROUS MEDIUM

R. P. Mathur[1]

Department of Mathematics

S.G.S.G. Govt. College, Nasirabad(Rajasthan)INDIA-305601

Nagendra Kumar[2]

Department of Mathematics

H.V.M. P.G. College, Raisi, Haridwar(Uttarakhand)INDIA-247671

ABSTRACT

The instability of the plane interface between two Oldroydian viscoelastic superposed fluids through porous medium in the presence of uniform rotation and variable magnetic field is discussed. The magnetic field, the viscosity and the density are assumed to be exponentially varying. For stable density stratification, the system is found to be stable for disturbances of all wave numbers. The magnetic field stabilizes the potentially unstable stratification for small wave-length perturbations which are otherwise unstable. The long wavelength perturbations remain unstable and are not stabilized by magnetic field. Rotation does not affect the stability or instability, as such, of stratification.

Keywords : Rayleigh-Taylor instability, Oldroydian fluid, uniform rotation, porous medium.

1. INTRODUCTION

A detailed account of the instability of the plane interface between two Newtonian fluids under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [4]. Bhatia [1] has considered the Rayleigh-Taylor instability of two viscous superposed conducting fluids in the presence of a uniform horizontal magnetic field. Bhatia and Steiner [3] have studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation and have found that the rotation has a destabilizing effect in contrast to the stabilizing effect on Newtonian fluid.

The problem in porous medium is of importance in soil, ground water hydrology and in atmosphere. When the fluid slowly percolates through the pores of a macroscopically homogeneous and isotropic porous medium, the gross effect is represented by Darcy's law according to which the usual viscous term in the equations of fluid motion is replaced by the resistance term $-(\mu / k_1)\vec{v}$, where μ is the viscosity of the fluid, k_1 the permeability of the medium and $\vec{V}$ the filter velocity of the fluid. Vest and Arpaci [11] have studied the stability of a horizontal layer of Maxwell's viscoelastic fluid heated from below. Sharma [10] has studied the instability of the plane interface between two Oldroydian viscoelastic superposed conducting fluids in the presence of a uniform magnetic field. Generally, the magnetic field has a stabilizing effect on the stability, but there are a few exceptions. For example, Kent [6] has studied the effect of a horizontal magnetic field that varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be unstable. Lapwood [8] has studied the stability of convective flow in hydromagnetics in a porous medium using Rayleigh's procedure. Bhatia and Mathur [2] have studied the Rayleigh-Taylor instability of two superposed Oldroydian fluids in a uniform vertical magnetic field through porous medium. The magnetic field stabilizes the unstable configuration for wave number band k>k* in which system is unstable in the absence of magnetic field. It is also found that the viscosity, viscoelasticity and medium porosity have stabilizing influence while elasticity and medium permeability have destabilizing influence.

Since viscoelastic fluids play an important role in polymers and in the electro-chemical industry, the studies of waves and stability in different viscoelastic fluids dynamical configuration has been carried out by several researchers in the recent past. Jukes [5] investigated the Rayleigh-Taylor instability problem in MHD with finite conductivity and found that finite conductivity introduces new and unexpected modes. Sengupta and Basak [9] have studied the instability of the plane interface separating two superposed viscoelastic (Maxwell) conducting fluids in a uniform vertical magnetic field. Numerically it is found that both viscosity and viscoelasticity of fluid have stabilizing influence while permeability of porous medium has mostly destabilizing effect on the growth rate of unstable mode of disturbance. Kumar and Singh [7] have studied the instability of two rotating Maxwellian viscoelastic superposed fluids with variable magnetic field in the presence of porous medium.

Keeping in mind the importance of non-Newtonian fluids in modern technology and industries and owing to the importance of variable magnetic field, rotation and porous medium in chemical engineering and geophysics, the present paper attempts to study the instability of the plane interface separating two incompressible superposed rotating Oldroydian viscoelastic fluids in porous medium in presence of a variable magnetic field.

2. PERTURBATION EQUATIONS

Let $T_{ij}, \tau_{ij}, e_{ij}, \mu, \lambda, \lambda_0(<\lambda), p, \delta_{ij}, v_i, x_i$ and $\frac{d}{dt}$ denote respectively the total stress tensor, the shear stress tensor, the rate of strain tensor, the viscosity, the stress relaxation time, the strain retardation time, the isotropic pressure, the kronecker delta, the velocity vector, the position vector and the mobile operator. Then the Oldroydian visoelastic fluid is described by the constitutive relations

$$\Gamma_{ij} = -p\delta_{ij} + \tau_{ij} \quad \text{...(1)}$$

$$\left(1+\lambda\frac{d}{dt}\right)\tau_{ij} = 2\mu\left(1+\lambda_0\frac{d}{dt}\right)e_{ij} \quad \text{...(2)}$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_1}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \quad \text{...(3)}$$

Above relations was proposed and studied by Oldroyd. Oldroyd also showed that many rheological equations of state of general validity, reduced to (1) – (3) when linearized. $\lambda_0 = 0$ yields the fluid to be Maxwellian, whereas $\lambda = 0 = \lambda_0$ gives the Newtonian viscous fluid.

We consider a static state in which an incompressible Oldroydian viscoelastic fluid is arranged in horizontal strata in porous medium and the pressure p and the density ⍰ are functions of the vertical coordinate z only. The system is acted on by a variable horizontal magnetic field $\vec{H}(H_0(z),0,0)$, a uniform rotation $\vec{\Omega}(0,0,\Omega)$ and a gravity force $\vec{g}(0,0,-g)$. The character of the equilibrium of this initial static state is determined as usual , by supposing that the system is slightly disturbed and then by following its further evolution.

Let $\vec{v}(u,v,w), \delta\rho, \delta p,$ and $\vec{h}(h_x, h_y, h_z)$ denote respectively the perturbations in fluid velocity (0,0,0), fluid density , fluid pressure p and the magnetic field $\vec{H}(H_0(z),0,0)$. Then the linearized hydromagnetic perturbation equations of rotating Oldroydian viscoelastic fluid in porous medium are

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial \vec{v}}{\partial t} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\delta p + \vec{g}\delta\rho + \frac{2\rho}{\varepsilon}\left(\vec{v}\times\vec{\Omega}\right) + \frac{\mu_e}{4\pi}\right.$$

$$\left.\left\{\left(\nabla\times\vec{h}\right)\times\vec{H} + \left(\nabla\times\vec{H}\right)\times\vec{h}\right\}\right] - \left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{\mu}{k_1}\vec{v}, \quad \text{...(4)}$$

$$\nabla.\vec{v} = 0 \quad \text{...(5)}$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t} = \left(\vec{H}.\nabla\right)\vec{v} - \left(\vec{v}.\nabla\right)\vec{H}, \quad \text{...(6)}$$

$$\nabla.\vec{h} = 0. \quad \text{...(7)}$$

$$\varepsilon\frac{\partial}{\partial t}\delta\rho = -w(D\rho), \quad \text{...(8)}$$

where ε is the medium porosity, μ_e the magnetic permeability and $D = \frac{d}{dz}$. Equation (8) results from the fact that the density of every particle remains unchanged as we follow it with its motion.

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y and t is given by

$$F(z)\exp\left(ik_x x + ik_y y + nt\right) \quad \text{...(9)}$$

where F(z) is some function of z, k_x and k_y are horizontal wave numbers, ($k^2 = k_x^2 + k_y^2$), and n may be complex, is the rate at which the system departs away from the equilibrium.

For perturbations of the form (9), (4)-(8) become,

$$\frac{\rho}{\varepsilon}(1+\lambda n)nu = -(1+\lambda n)ik_x\delta p + (1+\lambda n)\frac{\mu_e}{4\pi}h_z(DH) + (1+\lambda n)\frac{2\rho\Omega}{\varepsilon}v - (1+\lambda_0 n)\frac{\mu}{k_1}u \quad \text{...(10)}$$

$$\frac{\rho}{\varepsilon}(1+\lambda n)nv = -(1+\lambda n)ik_y\delta p + (1+\lambda n)\frac{\mu_e H}{4\pi}\left(ik_x h_y - ik_y h_x\right) - (1+\lambda n)\frac{2\rho\Omega}{\varepsilon}u$$

$$-(1+\lambda_0 n)\frac{\mu}{k_1}v \quad \text{...(11)}$$

$$\frac{\rho}{\varepsilon}(1+\lambda n)nw = -(1+\lambda n)[D\delta p + g\delta\rho] + (1+\lambda n)\frac{\mu_e H}{4\pi}\left[(ik_x h_z - Dh_x) - h_x\frac{(DH)}{H}\right] - (1+\lambda_0 n)\frac{\mu}{k_1}w$$ ...(12)

$ik_x u + ik_y v + Dw = 0$ ...(13)

$ik_x h_x + ik_y h_y + Dh_z = 0$ ...(14)

$\varepsilon\, nh_x = ik_x Hu - w(DH)$, ...(15)

$\varepsilon\, nh_y = ik_x Hv$, ...(16)

$\varepsilon\, nh_z = ik_x Hw$, ...(17)

ε n[illegible] [illegible] -=wD[illegible] . ...(18)

multiplying (10) and (11) by $-ik_x$, $-ik_y$ respectively, adding and using (13), (15)-(17), we obtain,

$$\frac{\rho}{\varepsilon}n(1+\lambda n)Dw = -k^2(1+\lambda n)\delta p - \frac{\mu}{k_1}(1+\lambda_0 n)Dw - \frac{2\rho\Omega}{\varepsilon}(1+\lambda n)\zeta$$

$$+(1+\lambda n)k_x k_y\frac{\mu_e H^2}{4\pi n\varepsilon}\zeta + \frac{(1+\lambda n)\mu_e Hk^2}{4\pi n\varepsilon}w(DH),$$ ...(19)

where ζ, the z-component of vorticity , is given by

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = ik_x v - ik_y u$$ ...(20)

Multyplying (10) and (11) by $-ik_y$, $+ik_x$, respectively , adding and using (13), (15)-(17), we obtain

$$\zeta = \frac{2(1+\lambda n)\Omega Dw}{(1+\lambda n)n + (1+\lambda_0 n)\frac{\nu\in}{k_1} + (1+\lambda n)\frac{k_x^2 V_A^2}{n}}$$ ...(21)

where $V_A^2 = \frac{\mu_e H^2}{4\pi\rho}$ is the square of the Alfv'en velocity and $\nu\left(=\frac{\mu}{\rho}\right)$ stands for kinematic viscosity.

Eliminating [illegible] p between12) and (19), using (21), we get after simplification

$$\left[(1+\lambda n)n + \frac{\nu\in}{k_1}\right]\left[D(\rho Dw) - k^2\rho w\right] + \frac{gk^2(1+\lambda n)}{n}(D\rho)w + 4(1+\lambda n)^2\Omega^2 n$$

$$\left[D\left\{\frac{\rho Dw}{(1+\lambda n)n^2 + (1+\lambda_0 n)\frac{n\nu\in}{k_1} + (1+\lambda n)k_x^2 V_A^2}\right\}\right]$$

$$+\frac{(1+\lambda n)\mu_e k_x^2}{4\pi n}\left[D\left(H^2 Dw\right)-k^2 H^2 w\right]=0 \qquad ...(22)$$

3. THE CASE OF EXPONENTIALLY VARYING DENSITY, VISCOSITY AND MAGNETIC FIELD

Assuming the stratifications in density, viscosity and magnetic field of the form

$$\rho=\rho_0 e^{\beta z}, \mu=\mu_0 e^{\beta z}, H^2=H_0^2 e^{\beta z}, \qquad ...(23)$$

where ρ_0, μ_0, H_0 and ▯ are constants. Equations(23) implies that the coefficient of kinematic viscosity and the Alfven velocity are constant every where. Using the stratification of the form (23), (22) transform to

$$\left[(1+\lambda n)n+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon}{k_1}+\frac{(1+\lambda n)}{n}V_A^2 k_x^2+\frac{4(1+\lambda n)^2\Omega^2 n}{(1+\lambda n)n^2+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon n}{k_1}+(1+\lambda n)k_x^2 V_A^2}\right]D^2 w$$

$$+\left[(1+\lambda n)n+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon}{k_1}+\frac{(1+\lambda n)}{n}V_A^2 k_x^2+\frac{4(1+\lambda n)^2\Omega^2 n}{(1+\lambda n)n^2+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon n}{k_1}+(1+\lambda n)k_x^2 V_A^2}\right]\beta Dw$$

$$-\left[(1+\lambda n)n+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon}{k_1}+\frac{(1+\lambda n)}{n}V_A^2 k_x^2-\frac{g\beta(1+\lambda n)}{n}\right]k^2 w=0 \qquad ...(24)$$

where $\upsilon_0=\frac{\mu_0}{\rho_0}$ and $V_A^2=\frac{\mu_e H_0^2}{4\pi\rho_0}$ are constant.

The general solution of (24) is

$$w=A_1 e^{q_1 z}+A_2 e^{q_2 z} \qquad ...(25)$$

Where A_1, A_2 are two arbitrary constants and q_1, q_2 are two roots of the equation

$$\left[(1+\lambda n)n+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon}{k_1}+\frac{(1+\lambda n)}{n}V_A^2 k_x^2+\frac{4(1+\lambda n)^2\Omega^2 n}{(1+\lambda n)n^2+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon n}{k_1}+(1+\lambda n)k_x^2 V_A^2}\right]q^2$$

$$+\left[(1+\lambda n)n+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon}{k_1}+\frac{(1+\lambda n)}{n}V_A^2 k_x^2+\frac{4(1+\lambda n)^2\Omega^2 n}{(1+\lambda n)n^2+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon n}{k_1}+(1+\lambda n)k_x^2 V_A^2}\right]\beta q$$

$$-\left[(1+\lambda n)n+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon}{k_1}+\frac{(1+\lambda n)}{n}V_A^2k_x^2-\frac{g\beta(1+\lambda n)}{n}\right]k^2=0 \qquad ...(26)$$

If the fluid is supposed to be confined between two rigid planes at z=0 and z=d, then the vanishing of w at z=0 is satisfied by the choice

$$w=A\left(e^{q_1 z}-e^{q_2 z}\right), \qquad ...(27)$$

while the vanishing of w at z = d requires

$$\exp(q_1-q_2)\,d=1, \qquad ...(28)$$

which imply that

$(q_1 - q_2)\, d = 2\, im$⍰ , ...(29)

where m is an integer.

Equation (28) gives

$$\begin{aligned}
q_{1,2}=&\left[\frac{-\beta}{2}\left\{(1+\lambda n)n+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon}{k_1}+\frac{(1+\lambda n)}{n}V_A^2k_x^2+\frac{4(1+\lambda n)^2\Omega^2 n}{(1+\lambda n)n^2+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon n}{k_1}+(1+\lambda n)k_x^2V_A^2}\right\}\right.\\
&\pm\frac{1}{2}\left[\beta^2\left\{(1+\lambda n)n+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon}{k_1}+\frac{(1+\lambda n)}{n}V_A^2k_x^2+\frac{4(1+\lambda n)^2\Omega^2 n}{(1+\lambda n)n^2+(1+\lambda_0 n)\frac{n\upsilon_0\varepsilon}{k_1}+(1+\lambda n)k_x^2V_A^2}\right\}^2\right.\\
&+4k^2\left\{(1+\lambda n)n+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon}{k_1}+\frac{(1+\lambda n)}{n}V_A^2k_x^2+\frac{4(1+\lambda n)^2\Omega^2 n}{(1+\lambda n)n^2+(1+\lambda_0 n)\frac{n\upsilon_0\varepsilon}{k_1}+(1+\lambda n)k_x^2V_A^2}\right\}\\
&\left.\left.\left\{(1+\lambda n)n+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon}{k_1}+\frac{(1+\lambda n)}{n}V_A^2k_x^2-\frac{g\beta(1+\lambda n)}{n}\right\}\right]^{\frac{1}{2}}\right]\times\\
&\left[(1+\lambda n)n+(1+\lambda_0 n)\frac{\upsilon_0\varepsilon}{k_1}+\frac{(1+\lambda n)}{n}V_A^2k_x^2+\frac{4(1+\lambda n)^2\Omega^2 n}{(1+\lambda n)n^2+(1+\lambda_0 n)\frac{n\upsilon_0\varepsilon}{k_1}+(1+\lambda n)k_x^2V_A^2}\right]^{-1} \qquad ...(30)
\end{aligned}$$

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Inserting the values of q_1,q_2 from (30) in equation (29), and simplifying, we obtain

$$\sum_{i=0}^{6} A_i n^i = 0 \qquad \text{...(31)}$$

Where

A_6 = A□^{2}

A_5 = 2A□ $\left(1+\lambda_0\upsilon_0\frac{\varepsilon}{k_1}\right)$

$$A_4 = A\left[1+2\lambda^2V_A^2k_x^2+2\lambda\upsilon_0\frac{\varepsilon}{k_1}+2\lambda_0\upsilon_0\frac{\varepsilon}{k_1}+\lambda_0^2\upsilon_0^2\right]+B\lambda^2\Omega^2-C\lambda^2$$

$$A_3 = 2A\left[\upsilon_0\frac{\varepsilon}{k_1}\left(1+\lambda_0+\lambda\lambda_0V_A^2k_x^2\right)+\lambda V_A^2k_x^2+\lambda_0\upsilon_0^2\frac{\varepsilon^2}{k_1^2}\right]+8B\lambda\Omega^2-C\left(1+\lambda+\lambda\lambda_0\upsilon_0\frac{\varepsilon}{k_1}\right)$$

$$A_2 = A\left[\frac{\upsilon_0^2\varepsilon^2}{k_1^2}+2k_x^2V_A^2\left(1+\frac{\lambda\upsilon_0\varepsilon}{k_1}+\lambda_0\upsilon_0\frac{\varepsilon}{k_1}\right)+\lambda^2k_x^4V_A^4\right]+B\Omega^2-C\left(1+\lambda^2V_A^2k_x^2+\upsilon_0\frac{\varepsilon}{k_1}(\lambda+\lambda_0)\right)$$

$$A_1 = 2AV_A^2k_x^2\left[\lambda V_A^2k_x^2+\upsilon_0\frac{\varepsilon}{k_1}\right]-C\left(\upsilon_0\frac{\varepsilon}{k_1}+2\lambda V_A^2k_x^2\right)$$

$$A_0 = k_x^2V_A^2\left(k_x^2V_A^2A-C\right), \qquad \text{...(32)}$$

where, we have put

A =□^{2}d^2 + 4m^2□^{2} + 4k^{2}d^2 ...(33)

B= □^{2}d^2 + 4m^2□^{2} ...(34)

C = 4 k^{2}d^{2}g□□ □
□ □ □ □□□□□□□□ □

Equation (31) is the dispersion relation studying the effect of rotation and the variable (exponential) horizontal magnetic field on the stability of stratified (exponentially varying density, viscosity) Oldroydian viscoelastic fluid in porous medium.

4. DISCUSSION

For stable stratification □ < 0□(31) does not have any change of sign and so has no positive root of n and the system is always stable for disturbances of all wave numbers.

For unstable stratification and if

$$V_A^2 < \frac{4g\beta k^2}{\left(\beta^2+\frac{4m^2\pi^2}{d^2}+4k^2\right)k_x^2}, \qquad \text{...(36)}$$

the constant term in (31) is negative, therefore , it has at least one positive real root and hence, the system is unstable for all wave numbers satisfying the inequality

$$k^2 < \frac{4g\beta \sec^2\theta}{V_A^2} - \frac{\beta^2 d^2 + 4m^2\pi^2}{4d^2} \qquad ...(37)$$

where θ is the angle between k_x and k (i.e. $k_x = k\cos\theta$).

If $\beta > 0$ (unstable stratification) and also

$$V_A^2 > \frac{4g\beta k^2}{\left(\beta^2 + \frac{4m^2\pi^2}{d^2} + 4k^2\right)k_x^2} \qquad ...(38)$$

then (31) has no positive root and so the system is stable.

Thus , for unstable density stratification and magnetic field such that

$$V_A^2 < \frac{4g\beta k^2}{\left(\beta^2 + \frac{4m^2\pi^2}{d^2} + 4k^2\right)k_x^2}, \qquad ...(39)$$

the system is unstable for all wave numbers satisfying

$$k^2 < \frac{g\beta \sec^2\theta}{V_A^2} - \frac{\beta^2 d^2 + 4m^2\pi^2}{4d^2} \qquad ...(40)$$

Also, it is clear from (31) that rotation does not affect the stability or instability, as such of stratification.

REFERENCES

1. **Bhatia, P.K.,** Rayleigh-Taylor instability of two viscous superposed conducting fluids, Nuovo Cimento, 19B (1974), 161 .

2. **Bhatia, P.K and Mathur, R. P.,** Instability of viscoelastic superposed fluids in a vertical magnetic field through porous medium, Ganita Sandesh, 17(2) (2003), 21.

3. **Bhatia, P.K. and Steiner, J.M.,** Thermal instability of a viscoelastic fluid layer in hydromagnetics, J. Math. Anal. Appl., 41 (1973), 271.

4. **Chandrasekhar, S.**, Hydrodynamic and Hydromagnetic Stability, Dover publication, New York, (1981).

5. **Jukes, I.D.,** On the Rayleigh-Taylor problem in magnetohydrodynamics with finite resistivity, J. Fluid Mech., 16 (1963), 177.

6**. Kent, A.,** Instability of laminar flow of a magneto fluid, Phys. Fluid, 9 (1966), 1286.

7**. Kumar, P. and Singh, M.,** Instability of two rotating Maxwellian viscoelastic superposed fluids with variable magnetic field in porous medium, Ganita Sandesh , 21(2) (2007), 199.

8. **Lapwood, E.R.,** Convection of a fluid in a porous medium, Proc. Camb. Phil. Soc, 44 (1948), 508.

9**. Sengupta, P.R and Basak, P.,** Stability of two superposed viscoelastic (Maxwell) Fluids in a Vertical Magnetic Field, Indian J. Pure Appl. Math., 35(7) (2004), 905.

10. **Sharma, R.C.,** Instability of the plane interface between two viscoelastic superposed conducting fluids, J. Math. Phys. Sci.,12 (1978), 603.

11. **Vest, CM. and Arpaci, V.S.**, Overstability of a viscoelastic fluid layer heated from below, J. Fluid Mech., 36 (1969), 613.